DRIFTING EFFECT OF ELECTRONS ON THE FORMATION OF ION-ACOUSTIC SOLITONS IN A PLASMA WITH NEGATIVE IONS

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Abstract— Theoretical investigation of the propagation of ion acoustic solitons in a collisionless plasma consisting of positive ions, negative ions and electrons under the drifting effect of electrons is carried out. Both compressive and rarefactive ion-acoustic solitons for slow mode and only compressive ion-acoustic solitons for fast mode are found to exist depending on the drift velocity v_e' of electrons for different values of Q' ($= m_j / m_i$, negative to positive ion mass ratio) >1 .

Keywords— Drifting velocity, Ion-acoustic, Soliton, KdV, Negative ions, Solitary wave, Compressive and rearefactive.

1 INTRODUCTION

THE propagation of ion-acoustic solitary waves have been studied by many researchers both theoretically and experimentally. Washimi and Taniuti [1] were the first to study the propagation of ion-acoustic solitary waves in a cold plasma. Karpman and Kadmtsev [2] have discussed the two effects of nonlinearity and dispersion that give rise to the solitary waves in plasma. On the other hand, total nonlinearity of these waves were taken into account by Sagdeev [3] in his derivation of energy integral instead of the perturbative technique to study solitary waves. Ikezi et al.[4] have first experimentally discovered ion-acoustic solitons and double layers in plasma. But in most of the investigations, the finite electron inertia is neglected leading to the Boltzmann equation for isothermal electrons or non-isothermal electrons. However, the characters of solitons are found to be substantially changed with the introduction of the complete fluid equation for the electrons with initial drift motion rather than the simple Boltzmann distribution. Leven and Steinmann [5], Kalita et al. [6], Kalita and Devi [7], Kalita and Das [8] and Kalita and Das [9] have studied ion-acoustic solitary waves with the assumption of drift motion of the electrons in a simple composite plasma. In presence of electron inertia, Das [10] has investigated the effect of ion temperature on small amplitude ion acoustic solitons in a magnetized ion-beam plasma. Very recently Das and Karmakar [11] have studied the formation of dust ion acoustic solitons in a plasma with the electrons' drift velocity through the modified Korteweg – de Vries (mKdV) equation.

The role of negative ions, whose presence cannot be ignored, in the formation of ion-acoustic solitons in a plasma is another important topic. The existence of ion-acoustic solitons in presence of negative ions in a plasma has been discussed by Das [12], Watanabe [13], Verheest [14], Baboolal et al. [15], Kalita and Kalita [16], Kalita and Devi [7], Kalita and Das [8], Kalita and Das [9] and Kalita and Barman [17]. Tagare [18] has investigated the modified KdV soliton for isothermal electrons with warm positive and negative ions whereas Tagare and Reddy [19] have considered the same problem for non-thermal electrons. Tajiri and Toda [20] predicts by pseudo potential method that solitary waves of large amplitude even can exist in a multi-component plasma with negative ions.

The experimental works in the laboratory with appreciable percentage of negative ions was started by Geoller et al.[21]. Ludwig et al. [22], Nakamura et al. [23] have observed the rarefactive solitons with small amplitudes in a plasma with a significant percentage of negative ions. The modified KdV soliton has been experimentally observed by Nakamura and Tsukabayashi [24] and Nakamura [25]. Further, Nakamura [26] has observed experimentally in a multicomponent plasma with negative ions of finite ion temperature that the minimum amplitude for a positive pulse to become solitary waves is measured as a function of the density of negative ions.

The present paper deals with the propagation of ion acoustic solitary waves in a plasma consisting of positive ions with negative ions together with the drift motion of the electrons in one dimension. To study the propagation of ion acoustic solitary waves in multispecies plasma we have used perturbation method. The organization of the paper is as follows. In Sec. 2 the basic equations for propagation of ion acoustic waves in an unmagnetized plasma system. In Sec. 3 derivation of KdV equation and solitary wave solution and Section 4 is kept for results and discussion.

2 BASIC EQUATIONS

We consider a one dimensional collisionless plasma consisting of both positive and negative ions, together with the usual electrons as follows:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial t}\left(n_i v_i\right)=0 \qquad (1)$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{\partial \phi}{\partial x}=0 \qquad (2)$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}\left(n_e v_e\right)=0 \qquad (3)$$

$$\left(\frac{\partial}{\partial t}+v_e\frac{\partial}{\partial x}\right)v_e=\frac{1}{Q}\left(\frac{\partial\phi}{\partial x}-\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \qquad (4)$$

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}\left(n_j v_j\right)=0 \qquad (5)$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}=\frac{1}{Q'}\frac{\partial\phi}{\partial x} \qquad (6)$$

Again for charge imbalances these equations are to be combined by the Poisson equation

$$\frac{\partial^2\phi}{\partial x^2}=n_e-n_i+n_j \qquad (7)$$

where, i, j and e stand for positive ion, negative ion and electron respectively , $Q=\frac{m_e}{m_i}$ (= electron to ion mass ratio), $Q'=\frac{m_j}{m_i}$ (= negative to positive ion mass ratio).

We have normalized densities n_i, n_j and n_e by the unperturbed densities n_{e0}, time t by the inverse of the characteristic ion plasma frequency i.e., $\omega_{pi}^{-1}=\left(\frac{m_i}{4\pi n_{i0}e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_{De}=\left(\frac{k_bT_e}{4\pi n_{i0}e^2}\right)^{\frac{1}{2}}$, velocities by the ion-acoustic speed $C_s=\left(\frac{k_bT_e}{m_i}\right)^{\frac{1}{2}}$, and the potential ϕ by $\frac{k_bT_e}{e}$; k_b is the Boltzmann constant.

3 DERIVATION OF KORTEWEG-DE VRIES EQUATION AND ITS SOLUTION

To derive the KdV equation from the set of equations (1) to (7), we use the stretched variables

$$\xi=\varepsilon^{\frac{1}{2}}(x-Ut), \quad \tau=\varepsilon^{\frac{3}{2}}Ut \qquad (8)$$

such that $\frac{\partial}{\partial x}\equiv\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}$, $\frac{\partial}{\partial t}\equiv\varepsilon^{\frac{1}{2}}U\left(\varepsilon\frac{\partial}{\partial\tau}-\frac{\partial}{\partial\xi}\right)$

with phase velocity U of the wave and the following expansions of the flow variables in terms of the smallness parameter ε:

$$n_i=n_{i0}+\varepsilon n_{i1}+\varepsilon^2 n_{i2}+\ldots$$

$$n_j=n_{j0}+\varepsilon n_{j1}+\varepsilon^2 n_{j2}+\ldots$$

$$n_e=1+\varepsilon n_{e1}+\varepsilon^2 n_{e2}+\ldots$$

$$v_i=\varepsilon v_{i1}+\varepsilon^2 v_{i2}+\ldots \qquad (9)$$

$$v_j=\varepsilon v_{j1}+\varepsilon^2 v_{j2}+\ldots$$

$$v_e=v'_e+\varepsilon v_{e1}+\varepsilon^2 v_{e2}+\ldots$$

$$\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots$$

With the use of of the transformation (8) and the expansion (9) in the normalized set of equations (1) – (7), we get the expression for the phase velocity and the KdV equation as follows-

$$\frac{1}{1-Q(U-v'_e)^2}=\frac{r+Q'}{U^2Q'(1-r)}, \quad r=\frac{n_{d0}}{n_{i0}} \qquad (10)$$

This gives

$$U=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

where $a=\{(1-r)Q'+Q(r+Q')\}$
$b=-(r+Q')Qv'_e$
$c=(r+Q')\left(Qv'^2_e-1\right)$

and

$$\frac{\partial\phi_1}{\partial\tau}+p\phi_1\frac{\partial\phi_1}{\partial\xi}+q\frac{\partial^3\phi_1}{\partial\xi^3}=0 \qquad (11)$$

where $p=\frac{A}{2B}$ and $q=\frac{1}{2B}$

with $A=\frac{3n_{i0}}{U^4}+\frac{3Q(U-v'_e)^2-1}{\left\{1-Q(U-v'_e)^2\right\}^3}-\frac{3n_{j0}}{Q'^2U^4}$

and $B=\frac{n_{i0}}{U^2}+\frac{QU(U-v'_e)}{\left\{1-Q(U-v'_e)^2\right\}^2}+\frac{n_{j0}}{Q'U^2}$

It is observed that the nonlinear ion-acoustic solitons, in this model of plasma, exist when $|U-v'_e|\neq\frac{1}{\sqrt{Q}}$.

By using the transformation $\eta=\xi-V\tau$ (V is the velocity with which the solitary waves travel to the right), the solution of the KdV equation (11) can be obtained as

$$\phi_1=\frac{3V}{p}\sec h^2\left(\frac{1}{2}\sqrt{\frac{V}{q}}\eta\right)$$

where $\phi_0=\frac{3V}{p}$ is the amplitude and $\Delta=2\sqrt{\frac{q}{V}}$ is the width of the ion-acoustic soliton.

4 RESULTS AND DISCUSSION

In presence of negative ions under the drifting effect of the electrons in this model of plasma, both fast and slow modes are found to exist for $Q'>1$ subject to the sound mathematical condition $v'_e\leq\sqrt{\frac{(1-r)Q'+Q(r+Q')}{(1-r)QQ'}}$. In this multi-component plasma, both compressive and rarefactive ion-acoustic solitons for slow mode and only compressive ion-acoustic solitons for fast mode are found to exist with high amplitudes for small v_e'. The variations of phase velocities U for slow and fast ion acoustic solitons are shown in the figure 1. It is evident from figure that phase velocity is influenced both by electrons drift velocity and concentration of negative ions. With the increasing v_e', the phase velocity for fast mode is seen to vary almost parabolically (continuous curves) and for slow mode it is increases very sharply (dotted curves) but at the upper regime of v_e' for fixed $r = 0.4$ and for different values of $Q' = 20(1), 25(2), 30(3)$. The amplitude [Fig. 2(a)] of the slow compressive ion-acoustic solitons decreases sharply in the lower region of v_e' and slowly in the upper region of v_e' for fixed $V = 0.30$ and $Q' = 18.5$. The corresponding width [Fig. 2(b)] of slow compressive ion-acoustic solitons decreases convexly for the same parametric values. For $Q' > 1$, the amplitudes [Fig. 3(a)] of the compressive KdV solitons diminish to zero at the upper existence region of v_e' for fixed $V = 0.10$ and $Q' = 5$ for $r = 0.05(1), 0.10(2), 0.15(3), 0.20(4)$. But these solitons were non-existent in the investigation of Kalita and Das [8]. Of course, these solitons attain high amplitudes at the lower region of v_e' which are higher for higher r. Noticeably, the amplitudes of the compressive KdV solitons are found to be considerably greater for higher values of r in presence of the electrons' drift motion in contrast to the works (without electrons drift motion) of Kalita and Barman [17](figure 1). Interestingly, the corresponding widths of KdV solitons with ignorable difference [Fig. 3(b)] for $r = 0.05(1), 0.10(2), 0.15(3), 0.20(4)$ sharply and convexly diminish. On the other hand, the amplitude [Fig. 4(a)] of the fast compressive soliton decreases sharply in the lower existence region of Q' and very slowly in upper existence region of Q' for fixed $V = 0.10$ and $r = 0.05$ for different values of $v_e' = 5(1), 10(2), 15(3)$. But corresponding width [Fig. 4(a)] of the solitons are of constant magnitude. It is to be mentioned that the compressive fast ion-acoustic solitons exists [Fig. 5 (a)] only for higher values of $Q'(>1)$ and for very small $V = 0.002$ and its amplitude decreases as Q' increases. Further they are found to exist only in upper regime of Q' for very small V and for different values of $v_e' = 6(1), 8(2), 10(3)$. On the other hand, rarefactive fast ion-acoustic solitons exist [Fig. 5 (a)] only in the lower regime of $Q'(>1)$ for the same set of small V. However, the character of the fast rarefactive solitons changes to fast compressive solitons after certain Q'^{*} characterizing an uncountable region. The corresponding widths [Fig. 5(b)] of the fast (compressive and rarefactive) increases very slowly and linearly for fixed $V = 0.002$ and $r = 0.4$ for different values of $v_e' = 6(1), 8(2), 10(3)$.

5 FIGURES

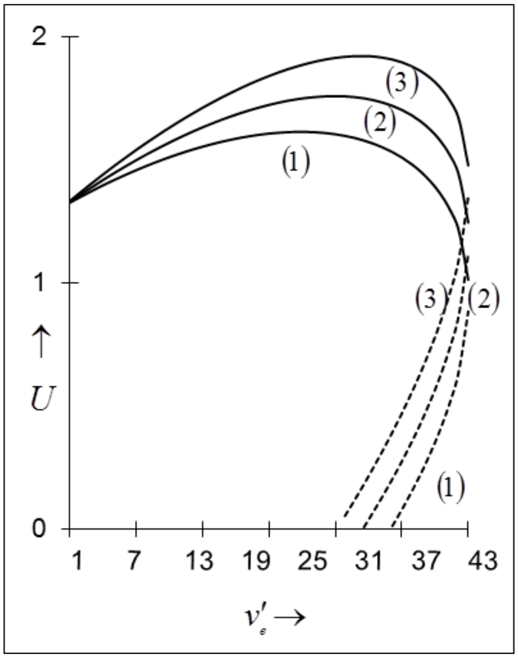

FIG.1. Phase velocity of slow (dotted curve) and fast (continuous curve) ion-acoustic solitons versus v_e' for fixed $r = 0.4$ for different values of $Q' = 20(1), 25(2), 30(3)$.

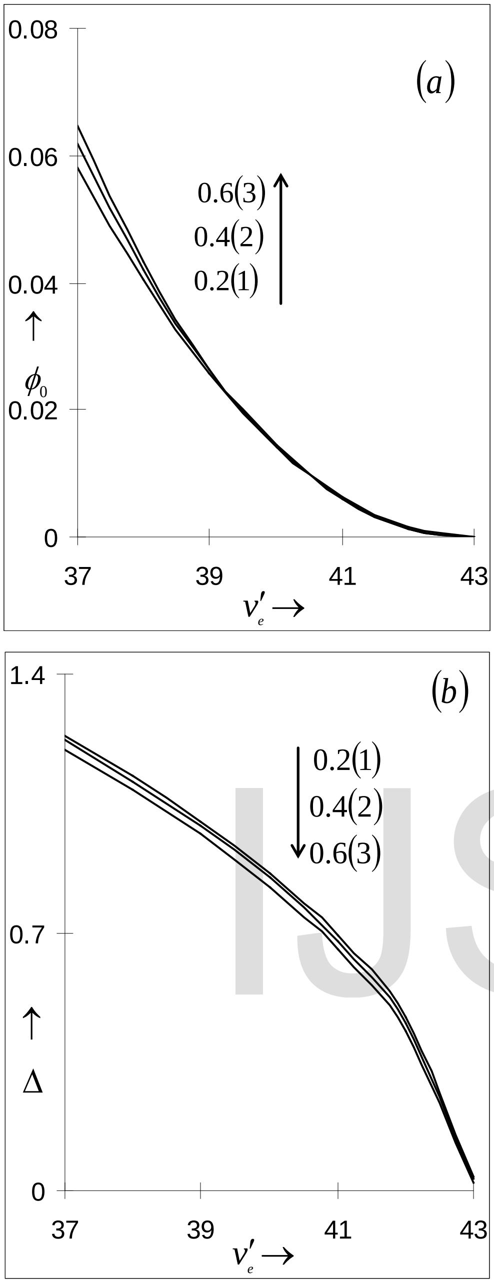

FIG.2. Amplitudes (a) and widths (b) of slow compressive ion-acoustic solitons versus v'_e for fixed $V = 0.30$ and $Q' = 18.5$ for different values of $r = 0.2(1), 0.4(2), 0.6(3)$.

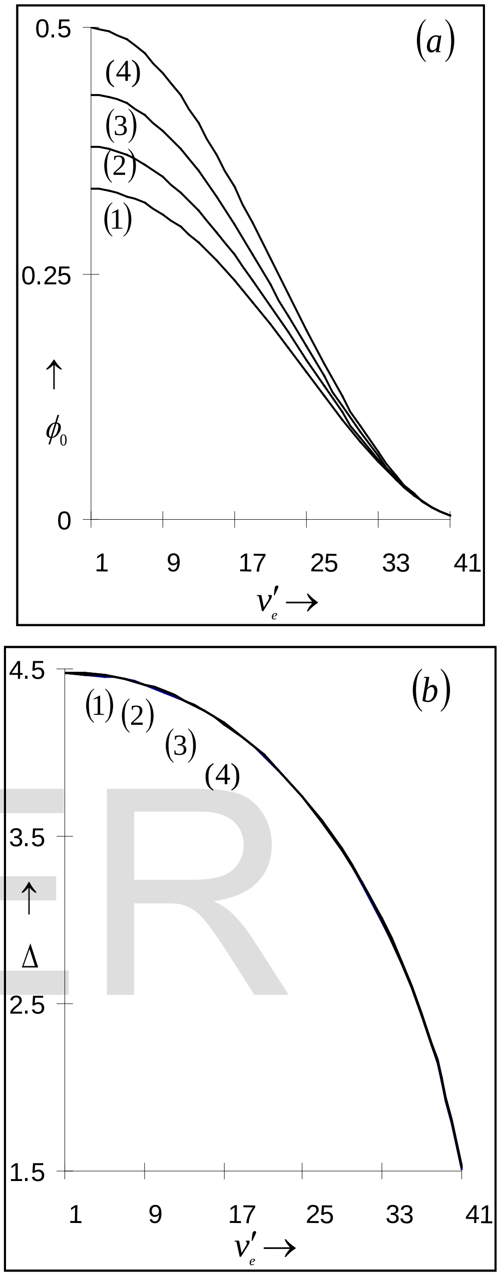

FIG.3. Amplitudes (a) and widths (b) of fast compressive ion-acoustic solitons versus v'_e for fixed $V = 0.10$ and $Q' = 5$ for different values of $r = 0.05(1), 0.10(2), 0.15(3), 0.20(4)$.

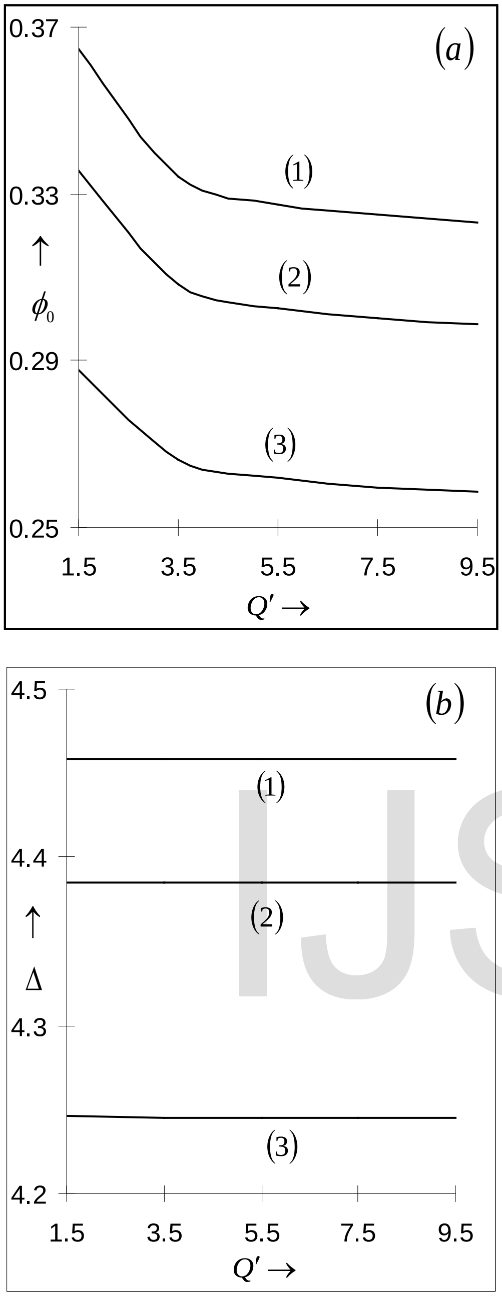

FIG.4. Amplitudes (a) and widths (b) of fast compressive ion-acoustic solitons versus Q' for fixed $V = 0.10$ and $r = 0.05$ for different values of $v_e' = 5(1), 10(2), 15(3)$.

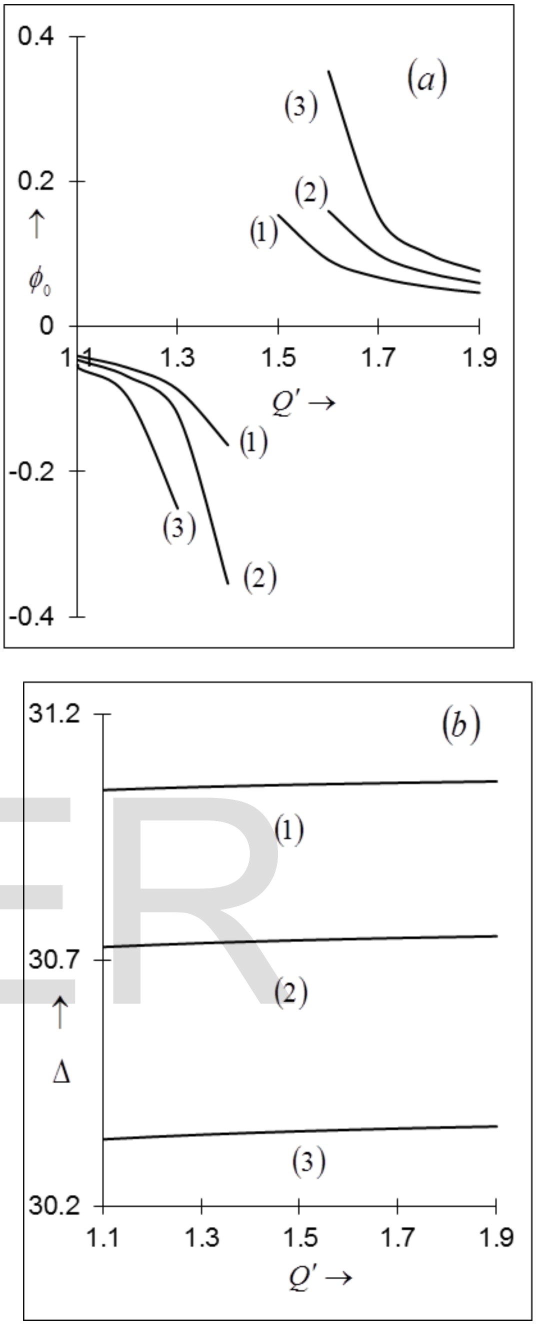

FIG.5. Amplitudes (a) and widths (b) of slow compressive and rarefactive ion-acoustic solitons versus Q' for fixed $V = 0.002$ and $r = 0.4$ for different values of $v_e' = 6(1), 8(2), 10(3)$.

REFERENCES

[1] H. Washimi and T. Taniuti, Propagation of Ion-Acoustic Solitary Waves of Small Amplitude, Phys. Rev. Lett. 17, 996 – 997 (1966).

[2] B. B. Kadomtsev and V. I. Karpman, Nonlinear Waves, Sov. Phys. Usp. **14**, 40- 60 (1971).

[3] R. Z. Sagdeev, Reviews of Plasma Physics (Consultants Bureau, New York, 1996), Vol. 3, p.33

[4] H. Ikezi, R. J. Tailor and D. R. Baker,Formation and Interaction of Ion-Acoustic Solitrons, Phys. Rev. Lett. **25**, 11 – 14 (1970).

[5] R. Leven and W. Steinmann, Drifting electron effects on strongly non-linear ion-acoustic solitons, Contr. Plasma Phys. **20,** 195 – 199(1980).

[6] B. C. Kalita, M. K. Kalita, and J. Chutia, Drifting effect of electrons on fully non-linear ion acoustic waves in a magnetoplasma, J.Phys. A Math, Gen. 19, 3559 – 3564 (1986).

[7] B. C. Kalita and N. Devi, Solitary waves in a warm plasma with negative ions and drifting effect of electrons, Phys. Fluids B ***5(2),*** *440 – 445 (1993).*

[8] B. C. Kalita and R. Das, A comparative study of modified Korteweg–de Vries (MKdV) and Korteweg–de Vries (KdV) solitons in plasmas with negative ions under the influence of electrons' drift motion, Plasmas **5**, 3588 – 3594 (1998).

[9] B. C. Kalita and R. Das, Modified Korteweg de Vries (MKdV) and Korteweg de Vries (KdV) Solitons in a Warm Plasma with Negative Ions and Electrons' Drift Motion J. Phys. Soc. Jpn. **72**, 2918-2924 (2002).

[10] R. Das, Effect of ion temperature on small-amplitude ion acoustic solitons in a magnetized ion-beam plasma in presence of electron inertia, Astrophys. Space Sci. 341, 543 – 549, (2012).

[11] R. Das and K. Karmakar, Modified Korteweg – de Vries solitons in a dusty plasma with electron inertia and drifting effect of electrons, Can. J. Phys, 91, 839 – 843 (2013).

[12] G. C. Das, G. C. Das, Ion-acoustic solutions and shock waves in multicomponent plasmas, Plasma Phys. 21, 257 – 266 (1979).

[13] S. Watanabe, Ion Acoustic Soliton in Plasma with Negative Ion, J. Phys. Soc. Jpn. **53**, 950 – 956 (1984).

[14] F. Verheest,.Ion-acoustic solitons in multi-component plasmas including negative ions at critical densities, J. Plasma Phys. 39, 71 – 79 (1988)

[15] S. Baboolal, R. Bharathram, and M. A. Hellberg,.Arbitrary-amplitude theory of ion-acoustic solitons in warm multi-fluid plasmas, J. Plasma Phys. **41**, 341 - 353(1989)

[16] B. C. Kalita and M. K. Kalita, Modified Korteweg–deVries solitons in a warm plasma with negative ions, Phys. Fluids B **2,** 674 – 676 (1995).

[17] B. C. Kalita and S. N. Barman, Solitons in a Warm Unmagnetized Plasma with Electron Inertia and Negative Ions, J. Phys. Soc. Jpn. **64**, 784 – 790 (1995).

[18] S. G. Tagare, Effect of ion temperature on ion-acoustic solitons in a two-ion warm plasma with adiabatic positive and negative ions and isothermal electrons, Plasma Phys. 36, 301 – 312 (1986).

[19] S. G. Tagare and R. V. Reddy, Effect of ionic temperature on ion-acoustic solitons in a two-ion warm plasma consisting of negative ions and non-isothermal electrons, Plasma Phys. Controlled Fusion 29, 671 – 676(1987).

[20] M. Tajiri and M. Toda, On Large Amplitude Ion Acoustic Solitons in Plasma with Negative Ions, J. Phys. Soc. Jpn. **54**, 19 – 22 (1985).

[21] S. Von. Goeller et al., Production of a Thermally Ionized Plasma with Negative Ions J. Appl. Phys. 37, 2519 (1966).

[22] G. O. Ludwig, J. L. Ferreira, and Y. Nakamura, Observation of Ion-Acoustic Rarefaction Solitons in a Multicomponent Plasma with Negative ions, Phys. Rev. Lett. **52,** 275 (1984).

[23] Y. Nakamura, L. Ferreira and G. O. Ludwig, Experiments on ion-acoustic rarefactive solitons in a multi-component plasma with negative ions, J. Plasma Phys. **33,** 237 - 248 (1985a).

[24] Y. Nakamura and I. Tsukabayashi, Modified Korteweg—de Vries ion-acoustic solitons in a plasma, J. Plasma Phys. 34, 401 – 415 (1985).

[25] Y. Nakamura, Nonlinear and Environmental Electromagnetics, ed. H. Kikuchi (Elsevier)p. 401 (1985).

[26] Y. Nakamura, Observation of large-amplitude ion acoustic solitary waves in a plasma, J. Plasma Phys. **38,** 461 – 471 (1987).